7.5 Applications from Science and Statistics

Work Revisited

$W = F d$ distance = dx

EXAMPLE 1 Finding the Work Done by a Force

Find the work done by the force $F(x) = \cos(\pi x)$ newtons along the x-axis from $x = 0$ meters to $x = 1/2$ meter.

$$\int_0^{1/2} \underbrace{\cos(\pi x)}_{F}\,\underbrace{dx}_{d}$$

$$\frac{1}{\pi}\sin(\pi x)\Big|_0^{1/2}$$

$$\frac{1}{\pi}\left(\sin\frac{\pi}{2} - \sin 0\right)$$

$$\frac{1}{\pi}(1 - 0) = \frac{1}{\pi}\,J$$

$J = N\,m$

EXAMPLE 2 Work Done Lifting

A leaky bucket weighs 22 newtons (N) empty. It is lifted from the ground at a constant rate to a point 20 m above the ground by a rope weighing 0.4 N/m. The bucket starts with 70 N (approximately 7.1 liters) of water, but it leaks at a constant rate and just finishes draining as the bucket reaches the top. Find the amount of work done

(a) lifting the bucket alone;

(b) lifting the water alone;

(c) lifting the rope alone;

(d) lifting the bucket, water, and rope together.

a) constant F → 22 N

d → 20 m

$W = Fd = 440\,Nm$ or $440\,J$

b) $F(x) = 70\left(-\frac{1}{20}x + 1\right)$

$= 70 - 3.5x$

$W(x) = \int_0^{20} (70 - 3.5x)\,dx$

$\left(70x - \frac{3.5}{2}x^2\right)\Big|_0^{20}$

$= 700\,Nm$ or $700\,J$

c) $F(x) = .4\,\frac{N}{m}\,(20 - x)$

$W(x) = \int_0^{20} .4(20 - x)\,dx$

$= 80\,Nm$ or $80\,J$

Total work = 440 + 700 + 80 = 1220 J